Properties of Exponents – Day 5

① $\dfrac{n}{n^{-\frac{7}{4}} \cdot m^{-\frac{1}{3}} \cdot n^{-1}} = \dfrac{n^{\frac{1}{1}}}{n^{-\frac{7}{4}} m^{-\frac{1}{3}} n^{-\frac{1}{1}}} = n^{\frac{1\cdot4}{1\cdot4}\frac{7}{4}} n^{\frac{1}{3}} m^{\frac{1\cdot4}{1\cdot4}} n^{\frac{4}{4}} = n^{\frac{4}{4}\frac{7}{4}} n^{\frac{1}{3}} m^{\frac{4}{4}} = \boxed{n^{\frac{15}{4}} m^{\frac{1}{3}}}$

② $\dfrac{y\, x^{\frac{3}{4}} \cdot x^{-1}}{(x^{\frac{2}{3}} y^{-1})^{-2}} = \dfrac{y^{\frac{1}{1}} x^{\frac{3}{4}} \cdot x^{-\frac{1}{1}}}{x^{-\frac{4}{3}} y^{\frac{2}{1}}} = \dfrac{y^{\frac{1}{1}} x^{\frac{3\cdot3}{4\cdot3}} x^{\frac{4\cdot4}{3\cdot4}}}{x^{\frac{1\cdot12}{1\cdot12}} y^{\frac{2}{1}}} = \dfrac{y^{\frac{1}{1}} x^{\frac{9}{12}} x^{\frac{16}{12}}}{x^{\frac{12}{12}} y^{\frac{2}{1}}} = \dfrac{y^{\frac{1}{1}} x^{\frac{25}{12}-\frac{12}{12}}}{x^{\frac{12}{12}} y^{\frac{2}{1}-\frac{1}{1}}} = \boxed{\dfrac{x^{\frac{13}{12}}}{y^{1}}}$

③ $\dfrac{b^{-\frac{1}{2}} a^{-\frac{1}{2}} b^{\frac{1}{2}}}{b^{-\frac{2}{3}}} = \dfrac{b^{\frac{2}{3}}}{b^{\frac{1}{2}} a^{\frac{1}{2}}} = \boxed{\dfrac{b^{\frac{2}{3}}}{a^{\frac{1}{2}}}}$

④ $\dfrac{x^{-1} y^{\frac{5}{3}}}{(x^{\frac{1}{4}} y^{\frac{1}{3}} \cdot x^{-\frac{3}{2}} y^{\frac{7}{4}})^{\frac{2}{3}}} = \dfrac{x^{-\frac{1}{1}} y^{\frac{5}{3}}}{x^{\frac{2}{12}} y^{\frac{2}{9}} x^{-\frac{6}{6}} y^{\frac{14}{12}}} = \dfrac{y^{\frac{5\cdot12}{3\cdot12}} x^{\frac{6\cdot2}{6\cdot2}}}{x^{\frac{2}{12}} y^{\frac{2\cdot4}{9\cdot4}} y^{\frac{14\cdot3}{12\cdot3}} x^{\frac{1\cdot12}{1\cdot12}}} = \dfrac{y^{\frac{60}{36}} x^{\frac{12}{12}}}{x^{\frac{2}{12}} y^{\frac{8}{36}} y^{\frac{42}{36}} x^{\frac{12}{12}}} = \dfrac{x^{\frac{12}{12}} y^{\frac{60}{36}-\frac{50}{36}}}{x^{\frac{14}{12}-\frac{12}{12}} y^{\frac{50}{36}}} = \dfrac{y^{\frac{10}{36}}}{x^{\frac{2}{12}}} =$

$= \boxed{\dfrac{y^{\frac{5}{18}}}{x^{\frac{1}{6}}}}$

⑤ $\left(\dfrac{uv}{v^{\frac{1}{2}} u^{-\frac{7}{4}} v^{\frac{3}{2}}}\right)^{-\frac{1}{2}} = \left(\dfrac{u^{\frac{1}{1}} v^{\frac{1}{1}}}{v^{\frac{1}{2}} u^{-\frac{7}{4}} v^{\frac{3}{2}}}\right)^{-\frac{1}{2}} = u^{-\frac{1}{2}} v^{-\frac{1}{2}} \cdot v^{-\frac{1}{4}} u^{\frac{7}{8}} v^{-\frac{3}{4}} = \dfrac{v^{\frac{1}{4}} v^{\frac{3}{4}}}{u^{\frac{7}{8}} u^{\frac{1\cdot4}{2\cdot4}} v^{\frac{1\cdot2}{2\cdot2}}} = \dfrac{v^{\frac{1}{4}} v^{\frac{3}{4}}}{u^{\frac{7}{8}} u^{\frac{4}{8}} v^{\frac{2}{4}}} = \dfrac{v^{\frac{4}{4}-\frac{2}{4}}}{u^{\frac{11}{8}} v^{\frac{2}{4}}} = \dfrac{v^{\frac{2}{4}}}{u^{\frac{11}{8}}} = \boxed{\dfrac{v^{\frac{1}{2}}}{u^{\frac{11}{8}}}}$

(6) $\dfrac{ab^2}{\left(ab^{\frac{7}{4}}\cdot a^{-\frac{4}{3}}\right)^{-\frac{1}{2}}} = \dfrac{a^{\frac{1}{1}}b^{\frac{2}{1}}}{\left(a^{\frac{1}{1}}b^{\frac{7}{4}}\cdot a^{-\frac{4}{3}}\right)^{-\frac{1}{2}}} = \dfrac{a^{\frac{1}{1}}b^{\frac{2}{1}}}{a^{-\frac{1}{2}}b^{-\frac{7}{8}}\cdot a^{\frac{4}{6}}} = \dfrac{a^{\frac{1\cdot6}{1\cdot6}}b^{\frac{2\cdot8}{1\cdot8}}a^{\frac{1\cdot3}{2\cdot3}}b^{\frac{7}{8}}}{a^{\frac{4}{6}}} = \dfrac{a^{\frac{6}{6}}b^{\frac{16}{8}}a^{\frac{3}{6}}b^{\frac{7}{8}}}{a^{\frac{4}{6}}} = a^{\frac{9-(-3)}{6}}b^{\frac{23}{8}} = \boxed{a^{\frac{5}{6}}b^{\frac{23}{8}}}$

(7) $\left(\dfrac{x^{-2}y^{-\frac{5}{4}}}{y^{\frac{1}{2}}x^{2}y}\right)^{\frac{3}{2}} = \left(\dfrac{x^{-\frac{2}{1}}y^{-\frac{5}{4}}}{y^{\frac{1}{1}}x^{\frac{1}{2}}y^{\frac{2}{1}}}\right)^{\frac{3}{2}} = \dfrac{x^{-\frac{6}{2}}y^{-\frac{15}{8}}}{y^{\frac{3}{2}}x^{\frac{3}{4}}y^{\frac{6}{2}}} = \dfrac{1}{x^{\frac{6\cdot2}{2\cdot2}}x^{\frac{3}{4}}y^{\frac{15}{8}}y^{\frac{3\cdot4}{2\cdot4}}y^{\frac{6\cdot4}{2\cdot4}}} = \dfrac{1}{x^{\frac{12}{4}}x^{\frac{3}{4}}y^{\frac{15}{8}}y^{\frac{12}{8}}y^{\frac{24}{8}}} = \boxed{\dfrac{1}{x^{\frac{15}{4}}y^{\frac{51}{8}}}}$

(8) $\dfrac{\left(y^{-\frac{2}{3}}\right)^{\frac{1}{3}}xy^{\frac{5}{4}}x^{2}}{\left(x^{-\frac{1}{3}}y^{\frac{1}{3}}\right)^{\frac{1}{2}}} = \dfrac{\left(y^{-\frac{2}{3}}\right)^{\frac{1}{3}}x^{\frac{1}{1}}y^{\frac{5}{4}}x^{\frac{2}{1}}}{\left(x^{-\frac{1}{3}}y^{\frac{1}{3}}\right)^{\frac{1}{2}}} = \dfrac{y^{-\frac{2}{9}}x^{\frac{1}{1}}y^{\frac{5}{4}}x^{\frac{2}{1}}}{x^{-\frac{1}{6}}y^{\frac{1}{6}}} = \dfrac{x^{\frac{1\cdot6}{1\cdot6}}x^{\frac{2\cdot6}{1\cdot6}}x^{\frac{1}{6}}y^{\frac{5\cdot9}{4\cdot9}}}{y^{\frac{2\cdot4}{9\cdot4}}y^{\frac{1\cdot6}{6\cdot6}}} = \dfrac{x^{\frac{6}{6}}x^{\frac{12}{6}}x^{\frac{1}{6}}y^{\frac{45}{36}}}{y^{\frac{8}{36}}y^{\frac{6}{36}}}$

$= \dfrac{x^{\frac{19}{6}}y^{\frac{45}{36}-\left(\frac{14}{36}\right)}}{y^{\frac{14}{36}}} = \boxed{x^{\frac{19}{6}}y^{\frac{31}{36}}}$

(9) $\dfrac{y^{\frac{1}{4}}\left(x^{-\frac{3}{2}}y^{2}\right)^{-\frac{1}{2}}\cdot y^{-2}}{x^{\frac{2}{3}}y^{\frac{4}{3}}} = \dfrac{y^{\frac{1}{4}}\left(x^{-\frac{3}{2}}y^{\frac{2}{1}}\right)^{-\frac{1}{2}}y^{-\frac{2}{1}}}{x^{\frac{2}{3}}y^{\frac{4}{3}}} = \dfrac{y^{\frac{1}{4}}x^{\frac{3}{4}}y^{-\frac{2}{2}}y^{-\frac{2}{1}}}{x^{\frac{2}{3}}y^{\frac{4}{3}}} = \dfrac{y^{\frac{1\cdot3}{4\cdot3}}x^{\frac{3\cdot3}{4\cdot3}}}{x^{\frac{2\cdot4}{3\cdot4}}y^{\frac{4\cdot4}{3\cdot4}}y^{\frac{2\cdot6}{2\cdot6}}y^{\frac{2\cdot12}{1\cdot12}}} = \dfrac{x^{\frac{3}{4}}y^{\frac{1}{4}}}{x^{\frac{2}{3}}}$

$= \dfrac{x^{\frac{9}{12}}y^{\frac{3}{12}}}{x^{\frac{8}{12}}y^{\frac{16}{12}}y^{\frac{12}{12}}y^{\frac{24}{12}}} = \dfrac{x^{\frac{9}{12}-\frac{8}{12}}y^{\frac{3}{12}}}{y^{\frac{52}{12}-\frac{3}{12}}} = \boxed{\dfrac{x^{\frac{1}{12}}}{y^{\frac{49}{12}}}}$

(10) $\left(\dfrac{m n \cdot m^{-\frac{3}{4}} n^{-2}}{m^{-\frac{7}{4}} n^{-\frac{7}{4}}}\right)^{\frac{3}{2}} = \left(\dfrac{m^{\frac{1}{1}} n^{\frac{1}{1}} m^{-\frac{3}{4}} n^{-\frac{2}{1}}}{m^{-\frac{7}{4}} n^{-\frac{7}{4}}}\right)^{\frac{3}{2}} = \dfrac{m^{\frac{3}{2}} n^{\frac{3}{2}} m^{-\frac{9}{8}} n^{-\frac{6}{2}}}{m^{-\frac{21}{8}} n^{-\frac{21}{8}}} = \dfrac{m^{\frac{3\cdot4}{2\cdot4}} n^{\frac{3\cdot4}{2\cdot4}} m^{\frac{21}{8}} n^{\frac{21}{8}}}{m^{\frac{9}{8}} n^{\frac{6\cdot4}{2\cdot4}}} = \dfrac{m^{\frac{12}{8}} n^{\frac{12}{8}} m^{\frac{21}{8}} n^{\frac{21}{8}}}{m^{\frac{9}{8}} n^{\frac{24}{8}}}$

$= \dfrac{m^{\frac{33}{8}-\frac{9}{8}} n^{\frac{33}{8}-\frac{24}{8}}}{m^{\frac{9}{8}} n^{\frac{24}{8}}} = m^{\frac{24}{8}} n^{\frac{9}{8}} = \boxed{m^{3} n^{\frac{9}{8}}}$

(11) $\left(\dfrac{u^{-1} v^{\frac{1}{2}}}{u^{-\frac{2}{3}} v^{\frac{3}{2}} u^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \left(\dfrac{u^{\frac{-1}{1}} v^{\frac{1}{2}}}{u^{-\frac{2}{3}} v^{\frac{3}{2}} u^{-\frac{1}{2}}}\right)^{\frac{1}{2}} = \dfrac{u^{-\frac{1}{2}} v^{\frac{1}{4}}}{u^{-\frac{2}{6}} v^{\frac{3}{4}} u^{-\frac{1}{4}}} = \dfrac{u^{\frac{2\cdot2}{6\cdot2}} u^{\frac{1\cdot3}{4\cdot3}} v^{\frac{1}{4}}}{u^{\frac{1}{2}\cdot\frac{6}{6}} v^{\frac{3}{4}}} = \dfrac{u^{\frac{4}{12}} u^{\frac{3}{12}} v^{\frac{1}{4}}}{u^{\frac{6}{12}} v^{\frac{3}{4}}} = \dfrac{u^{\frac{7}{12}-\frac{6}{12}} v^{\frac{1}{4}}}{u^{\frac{6}{12}} v^{\frac{3}{4}-\frac{1}{4}}}$

$= \dfrac{u^{\frac{1}{12}}}{v^{\frac{2}{4}}} = \boxed{\dfrac{u^{\frac{1}{12}}}{v^{\frac{1}{2}}}}$

(12) $\dfrac{(x^{2})^{\frac{3}{2}} \cdot x^{-1} y^{-\frac{3}{2}}}{x^{-3}} = \dfrac{(x^{\frac{2}{1}})^{\frac{3}{2}} x^{\frac{-1}{1}} y^{-\frac{3}{2}}}{x^{\frac{-3}{1}}} = \dfrac{x^{\frac{6}{2}} x^{-\frac{1\cdot2}{1\cdot2}} y^{-\frac{3}{2}}}{x^{\frac{-3\cdot2}{1\cdot2}}} = \dfrac{x^{\frac{6}{2}} x^{-\frac{2}{2}} y^{-\frac{3}{2}}}{x^{-\frac{6}{2}}} = \dfrac{x^{\frac{6}{2}} x^{\frac{6}{2}}}{x^{\frac{3}{2}} y^{\frac{3}{2}}} = \dfrac{x^{\frac{12}{2}-\frac{2}{2}}}{x^{\frac{3}{2}} y^{\frac{3}{2}}}$

$= \dfrac{x^{\frac{10}{2}}}{y^{3/2}} = \boxed{\dfrac{x^{5}}{y^{3/2}}}$

(13) $\dfrac{\left(y^{\frac{7}{4}}\right)^{-4}}{x y^{-1} x^{-1} y^{-\frac{4}{3}}} = \dfrac{\left(y^{\frac{7}{4}}\right)^{\frac{-4}{1}}}{\cancel{x}\cancel{x^{-1}} y^{-\frac{1}{1}} y^{-\frac{4}{3}}} = \dfrac{y^{-\frac{28}{4}}}{y^{-\frac{1}{1}} y^{-\frac{4}{3}}} = \dfrac{y^{\frac{1\cdot12}{1\cdot12}} y^{\frac{4\cdot4}{3\cdot4}}}{y^{\frac{28}{4}\cdot3}} = \dfrac{y^{\frac{12}{12}} y^{\frac{16}{12}}}{y^{\frac{74}{12}}} = \dfrac{y^{\frac{28}{12}}}{y^{\frac{84}{12}-\frac{28}{12}}} = \dfrac{1}{y^{\frac{56}{12}}} = \boxed{\dfrac{1}{y^{14/3}}}$

(14)

$$\frac{v^{2}}{u^{-2}\cdot u\,v^{\frac{3}{2}}} = \frac{v^{\frac{2}{1}}}{u^{-\frac{2}{1}}\,u^{\frac{1}{1}}\,v^{\frac{3}{2}}} = \frac{u^{\frac{2}{1}}\,v^{\frac{2\cdot 2}{1\cdot 2}}}{u^{1}\,v^{\frac{3}{2}}} = \frac{u^{\frac{2}{1}-\frac{1}{1}}\,v^{\frac{4}{2}-\frac{3}{2}}}{u^{\frac{1}{1}}\,v^{\frac{3}{2}}} = u^{\frac{1}{1}}\,v^{\frac{1}{2}} = \boxed{u\,v^{\frac{1}{2}}}$$

(15)

$$\frac{\left(x^{\frac{1}{3}}\left(x^{2}y^{\frac{1}{2}}\right)^{\frac{5}{4}}\right)^{-\frac{4}{3}}}{\left(x^{-\frac{1}{2}}y^{-2}\right)^{-\frac{1}{3}}} = \frac{\left(x^{\frac{1}{3}}\left(x^{\frac{2}{1}}y^{\frac{1}{2}}\right)^{\frac{5}{4}}\right)^{-\frac{4}{3}}}{\left(x^{-\frac{1}{2}}y^{-\frac{2}{1}}\right)^{-\frac{1}{3}}} = \frac{\left(x^{\frac{1}{3}}x^{\frac{10}{4}}y^{\frac{5}{8}}\right)^{-\frac{4}{3}}}{x^{\frac{1}{6}}y^{\frac{2}{3}}} = \frac{x^{-\frac{4}{9}}x^{-\frac{40}{12}}y^{-\frac{20}{24}}}{x^{\frac{1}{6}}y^{\frac{2}{3}}}$$

$$= \frac{1}{x^{\frac{1\cdot 6}{6\cdot 6}}x^{\frac{4\cdot 4}{9\cdot 4}}x^{\frac{40\cdot 3}{12\cdot 3}}y^{\frac{2\cdot 8}{3\cdot 8}}y^{\frac{20}{24}}} = \frac{1}{x^{\frac{6}{36}}x^{\frac{16}{36}}x^{\frac{120}{36}}y^{\frac{16}{24}}y^{\frac{20}{24}}} = \frac{1}{x^{\frac{142}{36}}y^{\frac{36}{24}}} = \boxed{\frac{1}{x^{\frac{71}{18}}y^{\frac{3}{2}}}}$$

(16)

$$\frac{x^{\frac{5}{3}}y^{\frac{1}{2}}}{x^{\frac{4}{3}}y^{-2}\cdot\left(x^{\frac{1}{2}}y^{\frac{5}{3}}\right)^{-\frac{7}{4}}\cdot x^{-1}y^{\frac{5}{3}}} = \frac{x^{\frac{5}{3}}y^{\frac{1}{2}}}{x^{\frac{4}{3}}y^{-\frac{2}{1}}\left(x^{\frac{1}{2}}y^{\frac{5}{3}}\right)^{-\frac{7}{4}}x^{-\frac{1}{1}}y^{\frac{5}{3}}} = \frac{x^{\frac{5}{3}}y^{\frac{1}{2}}}{x^{\frac{4}{3}}y^{-\frac{2}{1}}x^{-\frac{7}{8}}y^{-\frac{35}{12}}x^{-\frac{1}{1}}y^{\frac{5}{3}}} = \frac{x^{\frac{5\cdot 8}{3\cdot 8}}x^{\frac{7\cdot 3}{8\cdot 3}}x^{\frac{1\cdot 24}{1\cdot 24}}y^{\frac{1\cdot 6}{2\cdot 6}}y^{\frac{2\cdot 12}{1\cdot 12}}y^{\frac{35}{12}}}{x^{\frac{4\cdot 8}{3\cdot 8}}y^{\frac{5\cdot 4}{3\cdot 4}}}$$

$$= \frac{x^{\frac{40}{24}}x^{\frac{21}{24}}x^{\frac{24}{24}}y^{\frac{24}{12}}y^{\frac{35}{12}}y^{\frac{6}{12}}}{x^{\frac{32}{24}}y^{\frac{20}{12}}} = \frac{x^{\frac{85}{24}-\frac{33}{24}}y^{\frac{83}{12}-\frac{20}{12}}}{x^{\frac{32}{24}}y^{\frac{20}{12}}} = x^{\frac{53}{24}}y^{\frac{45}{12}} = \boxed{x^{\frac{53}{24}}y^{\frac{15}{4}}}$$