Unit 7: Representations of Exponential Relations

PRE-TEST

Determine if the sequence is geometric. If it is, find the common ratio.

1. 8, 7, 6, 5, ...

$\frac{7}{8}=\frac{7}{8}$ $\frac{6}{7}=\frac{6}{7}$ $\frac{5}{6}=\frac{5}{6}$

Not Geometric

2. 1, 2, 4, 8, ...

$\frac{2}{1}=2$ $\frac{4}{2}=2$ $\frac{8}{4}=2$

Geometric $r=2$

Find the recursive formula for each of the following:

3. 4, −12, 36, −108, ...

$a_1=4$

$\frac{-12}{4}=-3$ $\frac{36}{-12}=-3$ $\frac{-108}{36}=-3$

$a_n=a_{n-1}\cdot -3$

4. −2, −6, −18, −54, ...

$a_1=-2$

$\frac{-6}{-2}=3$ $\frac{-18}{-6}=3$ $\frac{-54}{-18}=3$

$a_n=a_{n-1}\cdot 3$

Find the explicit formula for each of the following:

5. −1, 6, −36, 216, ...

$a_n=a_1\cdot r^{n-1}$

$\frac{6}{-1}=-6$ $\frac{-36}{6}=-6$ $\frac{216}{-36}=-6$

$a_n=-1(-6)^{n-1}$

6. 3, 6, 12, 24, ...

$\frac{6}{3}=2$ $\frac{12}{6}=2$ $\frac{24}{12}=2$

$a_n=3(2)^{n-1}$

Given the following geometric sequences answer each of the following:

7. −1, −2, −4, −8, ...

A. Find the next three terms

$\frac{-2}{-1}=2$ $a_5=-8(2)=-16$

$\frac{-4}{-2}=2$ $a_6=-16(2)=-32$

$\frac{-8}{-4}=2$ $a_7=-32(2)=-64$

B. Find a_8

Explicit Formula

$a_n=-1(2)^{n-1}$

$a_8=-1(2)^{8-1}=-1(2)^7$

$=-1(128)=-128$

C. Find a_{10}

Explicit Formula

$a_{10}=-1(2)^{10-1}$

$=-1(2)^9$

$=-1(512)$

$=-512$

8. −4, −12, −36, −108, ...

A. Find the next three terms

$\frac{-12}{-4}=3$ $a_5=-108(3)$

-324

$\frac{-36}{-12}=3$ $a_6=-324(3)$

-972

$\frac{-108}{-36}=3$ $a_7=-972(3)$

-2916

B. Find a_8

Explicit Formula

$a_n=-4(3)^{n-1}$

$a_8=-4(3)^{8-1}=-4(3)^7$

$=-4(2187)$

$=-8748$

C. Find a_{12}

Explicit Formula

$a_{12}=-4(3)^{12-1}$

$=-4(3)^{11}$

$=-4(177147)$

$=-708588$

Using $a_n = a_1(r)^{n-1}$

Find the missing term or terms in each geometric sequence.

9. …, −2, ___, −72, … $a_2 = -12$	10. …, 2, ___, 18, … $a_2 = 6$
11. …, −1, ___, ___, −8, … $a_2 = -2$ $a_3 = -4$	12. …, −4, ___, ___, −32, … $a_2 = -8$ $a_3 = -16$
13. …, 1, ___, ___, ___, 81, … $a_2 = 3$ $a_3 = 9$ $a_4 = 27$	14. …, 1, ___, ___, ___, 16, … $a_2 = 2$ $a_3 = 4$ $a_4 = 8$
15. …, −3, ___, ___, ___, ___, −96, … $a_2 = -6$ $a_3 = -12$ $a_4 = -24$ $a_5 = -48$	16. …, 3, ___, ___, ___, ___, 23328; … $a_2 = 18$ $a_3 = 108$ $a_4 = 648$ $a_5 = 3888$
17. …, −1, ___, ___, ___, ___, ___, −15625, … $a_2 = -5$ $a_3 = -25$ $a_4 = -125$ $a_5 = -625$ $a_6 = -3125$	18. …, −4, ___, ___, ___, ___, ___, −2916, … $a_2 = -12$ $a_3 = -36$ $a_4 = -108$ $a_5 = -324$ $a_6 = -972$

Using $S_n = \frac{a_1(1-r^n)}{1-r}$

Evaluate each geometric series described.

19. $2 - 10 + 50 - 250 \ldots, n = 6$ $S_6 = -5208$	20. $-3 - 15 - 75 - 375 \ldots, n = 6$ $S_6 = -11718$
21. $-2 - 6 - 18 - 54 \ldots, n = 6$ $S_6 = -728$	22. $2 - 12 + 72 - 432 \ldots, n = 8$ $S_8 = -479890$
23. $\sum_{n=1}^{9}(5^{n-1})$ $S_9 = 488281$	24. $\sum_{n=1}^{8}(3 \cdot 5^{n-1})$ $S_8 = 292968$
25. $a_1 = 1, r = 5, n = 7$ $S_7 = 19531$	26. $a_1 = -4, r = -2, n = 8$ $S_8 = 340$

⑨ $\ldots, \underset{a_1}{-2}, \underset{a_2}{\_\_}, \underset{a_3}{-72}, \ldots$

$a_2 = -2 \cdot 6 = \boxed{-12}$

$a_n = a_1 (r)^{n-1}$

$-72 = -2(r)^{3-1}$

$\frac{-72}{-2} = \frac{-2(r)^2}{-2}$

$\sqrt{36} = \sqrt{r^2}$

$6 = r$

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⑩ $\ldots, \underset{a_1}{2}, \underset{a_2}{\_\_}, \underset{a_3}{18}, \ldots$

$a_2 = 2 \cdot 3 = \boxed{6}$

$a_n = a_1 (r)^{n-1}$

$18 = 2(r)^{3-1}$

$\frac{18}{2} = \frac{2(r)^2}{2}$

$\sqrt{9} = \sqrt{r^2}$

$3 = r$

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⑪ $\ldots, \underset{a_1}{-1}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{-8}, \ldots$

$a_2 = -1 \cdot 2 = \boxed{-2}$

$a_3 = -2 \cdot 2 = \boxed{-4}$

$a_n = a_1 (r)^{n-1}$

$-8 = -1(r)^{4-1}$

$\frac{-8}{-1} = \frac{-1(r)^3}{-1}$

$\sqrt[3]{8} = \sqrt[3]{r^3}$

$2 = r$

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⑫ $\ldots, \underset{a_1}{-4}, \underset{a_2}{\_\_}, \underset{a_3}{\_\_}, \underset{a_4}{-32}, \ldots$

$a_2 = -4 \cdot 2 = \boxed{-8}$

$a_3 = -8 \cdot 2 = \boxed{-16}$

$a_n = a_1 (r)^{n-1}$

$-32 = -4(r)^{4-1}$

$\frac{-32}{-4} = \frac{-4(r)^3}{-4}$

$\sqrt[3]{8} = \sqrt[3]{r^3}$

$2 = r$

(13) $\ldots, \underset{a_1}{1}, \_\_, \_\_, \_\_, \underset{a_5}{81}$

$a_2 = 1 \cdot 3 = \boxed{3}$

$a_3 = 3 \cdot 3 = \boxed{9}$

$a_4 = 9 \cdot 3 = \boxed{27}$

$a_n = a_1 (r)^{n-1}$

$81 = 1 (r)^{5-1}$

$\frac{81}{1} = \frac{1(r)^4}{1}$

$\sqrt[4]{81} = \sqrt[4]{r^4}$

$3 = r$

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(14) $\ldots, \underset{a_1}{1}, \_\_, \_\_, \_\_, \underset{a_5}{16}, \ldots$

$a_2 = 1 \cdot 2 = \boxed{2}$

$a_3 = 2 \cdot 2 = \boxed{4}$

$a_4 = 4 \cdot 2 = \boxed{8}$

$a_n = a_1 (r)^{n-1}$

$16 = 1 (r)^{5-1}$

$\frac{16}{1} = \frac{1(r)^4}{1}$

$\sqrt[4]{16} = \sqrt[4]{r^4}$

$2 = r$

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(15) $\ldots, \underset{a_1}{-3}, \_\_, \_\_, \_\_, \_\_, \underset{a_6}{-96}, \ldots$

$a_2 = -3 \cdot 2 = \boxed{-6}$

$a_3 = -6 \cdot 2 = \boxed{-12}$

$a_4 = -12 \cdot 2 = \boxed{-24}$

$a_5 = -24 \cdot 2 = \boxed{-48}$

$a_n = a_1 (r)^{n-1}$

$-96 = -3 (r)^{6-1}$

$\frac{-96}{-3} = \frac{-3(r)^5}{-3}$

$\sqrt[5]{32} = \sqrt[5]{r^5}$

$2 = r$

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(16) $\ldots, \underset{a_1}{3}, \_\_, \_\_, \_\_, \_\_, \underset{a_6}{23328}, \ldots$

$a_2 = 3 \cdot 6 = \boxed{18}$

$a_3 = 18 \cdot 6 = \boxed{108}$

$a_4 = 108 \cdot 6 = \boxed{648}$

$a_5 = 648 \cdot 6 = \boxed{3888}$

$a_n = a_1 (r)^{n-1}$

$23328 = 3 (r)^{6-1}$

$\frac{23328}{3} = \frac{3(r)^5}{3}$

$\sqrt[5]{7776} = \sqrt[5]{r^5}$

$6 = r$

(17) $\ldots, \underset{a_1}{-1}, \underset{a_2}{\_}, \underset{a_3}{\_}, \underset{a_4}{\_}, \underset{a_5}{\_}, \underset{a_6}{\_}, \underset{a_7}{-15625}, \ldots$

$a_n = a_1 (r)^{n-1}$

$-15625 = -1(r)^{7-1}$

$\frac{-15625}{-1} = \frac{-1(r)^6}{-1}$

$\sqrt[6]{15625} = \sqrt[6]{r^6}$

$5 = r$

$a_2 = -1 \cdot 5 = \boxed{-5}$

$a_3 = -5 \cdot 5 = \boxed{-25}$

$a_4 = -25 \cdot 5 = \boxed{-125}$

$a_5 = -125 \cdot 5 = \boxed{-625}$

$a_6 = -625 \cdot 5 = \boxed{-3125}$

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(18) $\ldots, \underset{a_1}{-4}, \underset{a_2}{\_}, \underset{a_3}{\_}, \underset{a_4}{\_}, \underset{a_5}{\_}, \underset{a_6}{\_}, \underset{a_7}{-2916}, \ldots$

$a_n = a_1 (r)^{n-1}$

$-2916 = -4(r)^{7-1}$

$\frac{-2916}{-4} = \frac{-4(r)^6}{-4}$

$\sqrt[6]{729} = \sqrt[6]{r^6}$

$3 = r$

$a_2 = -4 \cdot 3 = \boxed{-12}$

$a_3 = -12 \cdot 3 = \boxed{-36}$

$a_4 = -36 \cdot 3 = \boxed{-108}$

$a_5 = -108 \cdot 3 = \boxed{-324}$

$a_6 = -324 \cdot 3 = \boxed{-972}$

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(19) $2 - 10 + 50 - 250 \ldots, \; n = 6$

$a_1 = 2$

$r = \frac{-10}{2} = -5$

$n = 6$

$S_n = \frac{a_1 (1 - r^n)}{(1 - r)}$

$S_6 = \frac{2(1 - (-5)^6)}{1 - (-5)}$

$= \frac{2(1 - 15625)}{1 + 5}$

$= \frac{2(-15624)}{6} = \frac{-31248}{6} = \boxed{-5208}$

(20) $-3-15-75-375\ldots,\ n=6$

$a_1 = -3$

$r = \frac{-15}{-3} = 5$

$n = 6$

$$S_n = \frac{a_1(1-r^n)}{(1-r)}$$

$$S_6 = \frac{-3(1-(5)^6)}{1-5}$$

$$= \frac{-3(1-15625)}{-4}$$

$$= \frac{-3(-15624)}{-4}$$

$$= \frac{46872}{-4}$$

$$= \boxed{-11718}$$

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(21) $-2-6-18-54\ldots,\ n=6$

$a_1 = -2$

$r = \frac{-6}{-2} = 3$

$n = 6$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$= \frac{-2(1-(3)^6)}{1-3}$$

$$= \frac{-2(1-729)}{-2}$$

$$= \frac{-2(-728)}{-2}$$

$$= \frac{1456}{-2}$$

$$= \boxed{-728}$$

(22) $2 - 12 + 72 - 432 \ldots, \; n = 8$

$a_1 = 2$

$r = \frac{-12}{2} = -6$

$n = 8$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_8 = \frac{2(1 - (-6)^8)}{1 - (-6)}$$

$$= \frac{2(1 - 1679616)}{1 + 6}$$

$$= \frac{2(-1679615)}{7}$$

$$= \frac{-3359230}{7}$$

$$= \boxed{-479890}$$

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(23) $\sum_{n=1}^{9} (5^{n-1})$

$a_1 = 5^{1-1} = 5^0 = 1$

$r = 5$

$n = 9$

$$S_n = \frac{a_1(1 - r^n)}{1 - r}$$

$$S_9 = \frac{1(1 - (5)^9)}{1 - 5}$$

$$= \frac{1(1 - 1953125)}{4}$$

$$= \frac{1(-1953124)}{4}$$

$$= \frac{-1953124}{-4}$$

$$= \boxed{488281}$$

(24) $\sum_{n=1}^{8} (3 \cdot 5^{n-1})$

$a_1 = 3$

$r = 5$

$n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{3(1-(5)^8)}{1-5}$$

$$= \frac{3(1-390625)}{-4}$$

$$= \frac{3(-390624)}{-4}$$

$$= \frac{-1171872}{-4}$$

$$= \boxed{292968}$$

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(25) $a_1 = 1$; $r = 5$; $n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{1(1-(5)^7)}{1-5}$$

$$= \frac{1(1-78125)}{-4}$$

$$= \frac{1(-78124)}{-4}$$

$$= \frac{-78124}{-4}$$

$$= \boxed{19531}$$

(26) $a_1 = -4,\ r = -2,\ n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{-4(1-(-2)^8)}{1-(-2)}$$

$$= \frac{-4(1-256)}{1+2}$$

$$= \frac{-4(-255)}{3}$$

$$= \frac{1020}{3}$$

$$= \boxed{340}$$