Summations

Unit 7: Representations of Exponential Relations

Evaluate the related series of each sequence.

1. $3, 9, 27, 81$ $3+9+27+81 = \boxed{120}$	2. $-4, 24, -144, 864$ $-4+24-144+864 = \boxed{740}$

Evaluate each geometric series described.

3. $-1-2-4-8\ldots, n=7$ $S_7 = -127$	4. $-2+8-32+128\ldots, n=7$ $S_7 = -6554$
5. $\sum_{i=1}^{9} -4^{i-1}$ $S_9 = -87381$	6. $\sum_{n=1}^{7} (5)^{n-1}$ $S_7 = 19531$
7. $\sum_{k=1}^{7} -4(-2)^{k-1}$ $S_7 = -172$	8. $\sum_{m=1}^{7} -6^{m-1}$ $S_7 = -55987$
9. $a_1 = -3, a_n = 234375, r = -5$ $S_n = 195312$	10. $a_1 = 4, a_n = 2048, r = 2$ $S_n = 4092$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$S_n = \frac{a_1 - a_n \cdot r}{1-r}$

Evaluate each geometric series described.

11. $a_1 = -3, a_n = -234375, r = 5$ $S_n = -292968$	12. $a_1 = -3, a_n = -59049, r = 3$ $S_n = -88572$
13. $a_1 = 4, a_7 = 62500, r = 5$ $S_n = 78124$	14. $a_1 = 1, a_7 = 4096, r = 4$ $S_n = 5461$
15. $a_1 = 3, a_{10} = 59049, r = 3$ $S_n = 88572$	16. $a_1 = -4, a_8 = -512, r = 2$ $S_n = -1020$
17. $a_1 = -2, r = -4, n = 8$ $S_8 = 26214$	18. $a_1 = -3, r = 3, n = 10$ $S_{10} = -88572$
19. $a_1 = -4, r = -6, n = 7$ $S_7 = -159964$	20. $a_1 = 2, r = 3, n = 8$ $S_8 = 6560$

$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$

$S_n = \frac{a_1(1 - r^n)}{1 - r}$

③ $-1-2-4-8\ldots,\ n=7$

$a_1 = -1$

$r = \frac{-2}{-1} = 2$

$n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{-1(1-2^7)}{1-2}$$

$$S_7 = \frac{-1(1-128)}{-1}$$

$$S_7 = \frac{-1(-127)}{-1}$$

$$S_7 = \boxed{-127}$$

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④ $-2+8-32+128\ldots,\ n=7$

$a_1 = -2$

$r = \frac{8}{-2} = -4$

$n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{-2(1-(-4)^7)}{1-(-4)}$$

$$S_7 = \frac{-2(1-(-16384))}{1+4}$$

$$S_7 = \frac{-2(1+16384)}{5}$$

$$S_7 = \frac{-2(16385)}{5}$$

$$S_7 = -2(3277)$$

$$\boxed{S_7 = -6554}$$

⑤ $\sum_{i=1}^{9} -4^{i-1}$

$a_1 = -1$

$r = 4$

$n = 9$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_9 = \frac{-1(1-4^9)}{1-4}$$

$$S_9 = \frac{-1(1-262144)}{-3}$$

$$S_9 = \frac{-1(-262143)}{-3}$$

$$S_9 = \frac{262143}{-3}$$

$$\boxed{S_9 = -87381}$$

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⑥ $\sum_{n=1}^{7} (5)^{n-1}$

$a_1 = 1$

$r = 5$

$n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{1(1-5^7)}{1-5}$$

$$S_7 = \frac{1(1-78125)}{-4}$$

$$S_7 = \frac{1(-78124)}{-4}$$

$$S_7 = \frac{-78124}{-4}$$

$$\boxed{S_7 = 19531}$$

⑦ $\sum_{K=1}^{7} -4(-2)^{K-1}$

$a_1 = -4$

$r = -2$

$n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{-4(1-(-2)^7)}{1-(-2)}$$

$$S_7 = \frac{-4(1-(-128))}{1+2}$$

$$S_7 = \frac{-4(129)}{3}$$

$$S_7 = \frac{-516}{3}$$

$$\boxed{S_7 = -172}$$

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⑧ $\sum_{m=1}^{7} -6^{m-1}$

$a_1 = -1$

$r = 6$

$n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_7 = \frac{-1(1-6^7)}{1-6}$$

$$S_7 = \frac{-1(1-279936)}{-5}$$

$$S_7 = \frac{-1(-279935)}{-5}$$

$$S_7 = \frac{279935}{-5}$$

$$\boxed{S_7 = -55987}$$

⑨ $a_1 = -3,\ a_n = 234375,\ r = -5$

New equation for #9-16

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{-3 - 234375(-5)}{1 - (-5)}$$

$$S_n = \frac{-3 + 1171875}{1 + 5}$$

$$S_n = \frac{1171872}{6}$$

$$\boxed{S_n = 195312}$$

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⑩ $a_1 = 4,\ a_n = 2048,\ r = 2$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{4 - 2048(2)}{1 - 2}$$

$$S_n = \frac{4 - 4096}{-1}$$

$$S_n = \frac{-4092}{-1}$$

$$\boxed{S_n = 4092}$$

(11) $a_1 = -3,\ a_n = -234375,\ r = 5$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{-3 - (-234375)(5)}{1 - 5}$$

$$S_n = \frac{-3 - (-1171875)}{1 - 5}$$

$$S_n = \frac{-3 + 1171875}{-4}$$

$$S_n = \frac{1171872}{-4}$$

$$\boxed{S_n = -292968}$$

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(12) $a_1 = -3,\ a_n = -59049,\ r = 3$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{-3 - (-59049)(3)}{1 - 3}$$

$$S_n = \frac{-3 - (-177147)}{1 - 3}$$

$$S_n = \frac{-3 + 177147}{-2}$$

$$S_n = \frac{177144}{-2}$$

$$\boxed{S_n = -88572}$$

(13) $a_1 = 4,\ a_7 = 62500,\ r = 5$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{4 - 62500(5)}{1 - 5}$$

$$S_n = \frac{4 - 312500}{1 - 5}$$

$$S_n = \frac{-312496}{-4}$$

$$\boxed{S_n = 78124}$$

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(14) $a_1 = 1,\ a_7 = 4096,\ r = 4$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{1 - 4096(4)}{1 - 4}$$

$$S_n = \frac{1 - 16384}{1 - 4}$$

$$S_n = \frac{-16383}{-3}$$

$$\boxed{S_n = 5461}$$

(15) $a_1 = 3,\ a_{10} = 59049,\ r = 3$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{3 - 59049(3)}{1 - 3}$$

$$S_n = \frac{3 - 177147}{1 - 3}$$

$$S_n = \frac{-177144}{-2}$$

$$\boxed{S_n = 88572}$$

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(16) $a_1 = -4,\ a_8 = -512,\ r = 2$

$$S_n = \frac{a_1 - a_n \cdot r}{1 - r}$$

$$S_n = \frac{-4 - (-512)(2)}{1 - 2}$$

$$S_n = \frac{-4 - (-1024)}{1 - 2}$$

$$S_n = \frac{-4 + 1024}{1 - 2}$$

$$S_n = \frac{1020}{-1}$$

$$\boxed{S_n = -1020}$$

(17) $a_1 = -2,\ r = -4,\ n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{-2(1-(-4)^8)}{1-(-4)}$$

$$S_8 = \frac{-2(1-65536)}{1-(-4)}$$

$$S_8 = \frac{-2(-65535)}{1+4}$$

$$S_8 = \frac{131070}{5}$$

$$\boxed{S_8 = 26214}$$

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(18) $a_1 = -3,\ r = 3,\ n = 10$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_{10} = \frac{-3(1-3^{10})}{1-3}$$

$$S_{10} = \frac{-3(1-59049)}{1-3}$$

$$S_{10} = \frac{-3(-59048)}{1-3}$$

$$S_{10} = \frac{177144}{-2}$$

$$\boxed{S_{10} = -88572}$$

(19) $a_1 = -4,\quad r = -6,\quad n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_n = \frac{-4(1-(-6)^7)}{1-(-6)}$$

$$S_n = \frac{-4(1-(-279936))}{1-(-6)}$$

$$S_n = \frac{-4(1+279936)}{1+6}$$

$$S_n = \frac{-4(279937)}{1+6}$$

$$S_n = \frac{-1119748}{7}$$

$$\boxed{S_n = -159964}$$

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(20) $a_1 = 2,\quad r = 3,\quad n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_8 = \frac{2(1-3^8)}{1-3}$$

$$S_8 = \frac{2(1-6561)}{1-3}$$

$$S_8 = \frac{2(-6560)}{1-3}$$

$$S_8 = \frac{-13120}{-2}$$

$$\boxed{S_8 = 6560}$$