

## Summations

### Unit 6: Representations of Linear Relations

Evaluate the related series of each sequence.

1. 26, 34, 42, 50, 58  $26 + 34 + 42 + 50 + 58 = \boxed{210}$	2. 16, 23, 30, 37, 44, 51  $16 + 23 + 30 + 37 + 44 + 51 = \boxed{201}$
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Evaluate each arithmetic series described.

3. $43 + 52 + 61 + 70 \dots, n = 9$  $S_9 = \boxed{711}$	4. $7 + 15 + 23 + 31 \dots, n = 15$  $S_{15} = \boxed{945}$
5. $6 + 10 + 14 + 18 \dots, n = 13$  $S_{13} = \boxed{390}$	6. $17 + 26 + 35 + 44 \dots, n = 9$  $S_9 = \boxed{477}$
7. $\sum_{n=1}^{12} (4n - 9)$ $d = 4$ $n = 12$ $a_1 = 4(1) - 9$ $a_1 = 4 - 9$ $a_1 = \boxed{-5}$ $S_{12} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_{12} = \frac{12}{2} (2 \cdot (-5) + (12-1) \cdot 4)$ $S_{12} = 6(-10 + (11)(4))$ $S_{12} = 6(-10 + 44)$ $S_{12} = 6(34)$ $S_{12} = \boxed{204}$	8. $\sum_{i=1}^7 (3i + 5)$ $d = 3$ $n = 7$ $a_1 = 3(1) + 5$ $a_1 = 3 + 5$ $a_1 = \boxed{8}$ $S_7 = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_7 = \frac{7}{2} (2 \cdot 8 + (7-1) \cdot 3)$ $S_7 = 3.5(16 + (6)(3))$ $S_7 = 3.5(16 + 18)$ $S_7 = 3.5(34)$ $S_7 = \boxed{119}$
9. $\sum_{n=1}^{25} (9n - 10)$ $d = 9$ $n = 25$ $a_1 = 9(1) - 10$ $a_1 = 9 - 10$ $a_1 = \boxed{-1}$ $S_{25} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_{25} = \frac{25}{2} (2 \cdot (-1) + (25-1) \cdot 9)$ $S_{25} = 12.5(-2 + (24)(9))$ $S_{25} = 12.5(-2 + 216)$ $S_{25} = 12.5(214)$ $S_{25} = \boxed{2675}$	10. $\sum_{m=1}^{25} (9m - 16)$ $d = 9$ $n = 25$ $a_1 = 9(1) - 16$ $a_1 = 9 - 16$ $a_1 = \boxed{-7}$ $S_{25} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$ $S_{25} = \frac{25}{2} (2 \cdot (-7) + (25-1) \cdot 9)$ $S_{25} = 12.5(-14 + (24)(9))$ $S_{25} = 12.5(-14 + 216)$ $S_{25} = 12.5(202)$ $S_{25} = \boxed{2525}$

Evaluate each arithmetic series described.

11.  $a_1 = 37, a_n = 97, n = 7$

$$S_7 = \frac{n}{2} (a_1 + a_n)$$

$$S_7 = \frac{7}{2} (37 + 97)$$

$$S_7 = 3.5(134)$$

$$S_7 = 469$$

12.  $a_1 = 14, a_n = 38, n = 7$

$$S_7 = \frac{n}{2} (a_1 + a_n)$$

$$S_7 = \frac{7}{2} (14 + 38)$$

$$S_7 = 3.5(52)$$

$$S_7 = 182$$

13.  $a_1 = 46, a_n = 136, n = 11$

$$S_{11} = \frac{n}{2} (a_1 + a_n)$$

$$S_{11} = \frac{11}{2} (46 + 136)$$

$$S_{11} = 5.5(182)$$

$$S_{11} = 1001$$

14.  $a_1 = 10, a_n = 25, n = 6$

$$S_6 = \frac{n}{2} (a_1 + a_n)$$

$$S_6 = \frac{6}{2} (10 + 25)$$

$$S_6 = 3(35)$$

$$S_6 = 105$$

15.  $a_1 = -9, a_n = -87, n = 14$

$$S_{14} = \frac{n}{2} (a_1 + a_n)$$

$$S_{14} = \frac{14}{2} (-9 - 87)$$

$$S_{14} = 7(-96)$$

$$S_{14} = -672$$

16.  $a_1 = 8, a_n = 200, n = 25$

$$S_{25} = \frac{n}{2} (a_1 + a_n)$$

$$S_{25} = \frac{25}{2} (8 + 200)$$

$$S_{25} = 12.5(208)$$

$$S_{25} = 2600$$

17.  $a_1 = 14, d = 3, n = 11$

$$S_{11} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{11} = \frac{11}{2} (2 \cdot 14 + (11-1) \cdot 3)$$

$$S_{11} = 5.5(28 + (10)(3))$$

$$S_{11} = 5.5(28 + 30)$$

$$S_{11} = 5.5(58) = 319$$

18.  $a_1 = 42, d = 9, n = 10$

$$S_{10} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{10} = \frac{10}{2} (2 \cdot 42 + (10-1) \cdot 9)$$

$$S_{10} = 5(84 + (9)(9))$$

$$S_{10} = 5(84 + 81)$$

$$S_{10} = 5(165) = 825$$

19.  $a_1 = 7, d = 5, n = 7$

$$S_7 = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_7 = \frac{7}{2} (2 \cdot 7 + (7-1) \cdot 5)$$

$$S_7 = 3.5(14 + (6)(5))$$

$$S_7 = 3.5(14 + 30)$$

$$S_7 = 3.5(44) = 154$$

20.  $a_1 = -11, d = -5, n = 12$

$$S_{12} = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_{12} = \frac{12}{2} (2 \cdot -11 + (12-1) \cdot -5)$$

$$S_{12} = 6(-22 + (11)(-5))$$

$$S_{12} = 6(-22 - 55)$$

$$S_{12} = 6(-77) = -462$$

③ Information Needed:

$$d = 52 - (43) = 9$$

$$a_1 = 43$$

$$n = 9$$

Using the formula:

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{9}{2} (2 \cdot 43 + (9-1) \cdot 9)$$

$$S_n = 4.5 (86 + (8)(9))$$

$$S_n = 4.5 (86 + 72)$$

$$S_n = 4.5 (158)$$

$$S_n = 711$$

④ Information Needed:

$$d = 15 - (7) = 8$$

$$a_1 = 7$$

$$n = 15$$

Using the formula:

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{15}{2} (2 \cdot 7 + (15-1) \cdot 8)$$

$$S_n = 7.5 (14 + (14)(8))$$

$$S_n = 7.5 (14 + 112)$$

$$S_n = 7.5 (126)$$

$$S_n = 945$$

⑤ Information Needed:

$$d = 10 - (6) = 4$$

$$a_1 = 6$$

$$n = 13$$

Using the formula:

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{13}{2} (2 \cdot 6 + (13-1) \cdot 4)$$

$$S_n = 6.5 (12 + (12)(4))$$

$$S_n = 6.5 (12 + 48)$$

$$S_n = 6.5 (60)$$

$$S_n = 390$$

⑥ Information Needed:

$$d = 26 - (17) = 9$$

$$a_1 = 17$$

$$n = 9$$

Using the formula:

$$S_n = \frac{n}{2} (2 \cdot a_1 + (n-1) \cdot d)$$

$$S_n = \frac{9}{2} (2 \cdot 17 + (9-1) \cdot 9)$$

$$S_n = 4.5 (34 + 8(9))$$

$$S_n = 4.5 (34 + 72)$$

$$S_n = 4.5 (106)$$

$$S_n = 477$$

Using the formula:  
 $(b \cdot (n-1) + a_1) \cdot \frac{n}{2} = S_n$   
 $(17 + (n-1) \cdot 9) \cdot \frac{n}{2} = 477$   
 $(17 + 9n - 9) \cdot \frac{n}{2} = 477$   
 $(8 + 9n) \cdot \frac{n}{2} = 477$   
 $(8n + 9n^2) \cdot \frac{1}{2} = 477$   
 $8n + 9n^2 = 954$   
 $9n^2 + 8n - 954 = 0$   
 $n = 10$

Information needed:  
 $b = 26 - (17) = 9$   
 $a_1 = 17$   
 $n = 9$

Using the formula:  
 $(b \cdot (n-1) + a_1) \cdot \frac{n}{2} = S_n$   
 $(17 + (n-1) \cdot 9) \cdot \frac{n}{2} = 477$   
 $(17 + 9n - 9) \cdot \frac{n}{2} = 477$   
 $(8 + 9n) \cdot \frac{n}{2} = 477$   
 $8n + 9n^2 = 954$   
 $9n^2 + 8n - 954 = 0$   
 $n = 10$

Information needed:  
 $b = 26 - (17) = 9$   
 $a_1 = 17$   
 $n = 9$